

Consider the region defined by $y \leq x+1$, $y \geq 2x-3$ and $x \geq 1$.

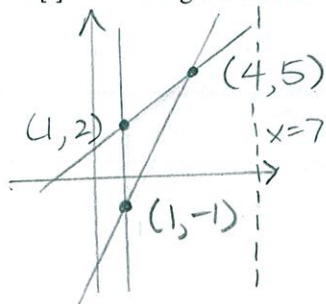
$$\begin{aligned} x+1 &= 2x-3 \\ 4 &= x \rightarrow y=5 \end{aligned}$$

SCORE: ____ / 12 PTS

[a] If the region is revolved around the line $x=7$,
 $x=y-1$, $x=\frac{1}{2}(y+3)$

write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

[i] using the disk or washer method (**NOTE: You do NOT need to simplify your integrand.**)



$$\begin{aligned} & \int_{2\frac{1}{2}}^5 \pi \left[\left(7 - (y-1) \right)^2 - \left(7 - \frac{1}{2}(y+3) \right)^2 \right] dy \\ & + \int_{-1}^2 \pi \left[\left(7 - 1 \right)^2 - \left(7 - \frac{1}{2}(y+3) \right)^2 \right] dy \end{aligned}$$

[ii] using the shell method (**NOTE: You do NOT need to simplify your integrand.**)

$$2\pi \int_1^4 (7-x)(x+1-(2x-3)) dx$$

① POINT EACH EXCEPT AS NOTED

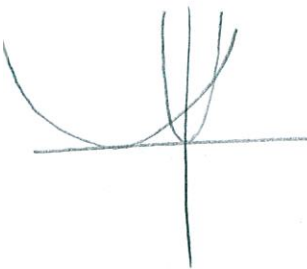
[b] Suppose the region is the base of a solid. Cross sections perpendicular to the x -axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\pi}{8} \int_1^4 (x+1-(2x-3))^2 dx$$

Find the area between the curves $y = 4x^2$ and $y = (x+3)^2$ over the interval $-2 \leq x \leq 2$.

SCORE: ____ / 6 PTS

NOTE: The answer is NOT 20 NOR 4.



$$4x^2 = (x+3)^2$$

$$4x^2 = x^2 + 6x + 9$$

$$3x^2 - 6x - 9 = 0$$

$$3(x-3)(x+1) = 0$$

$$x = 3, -1$$

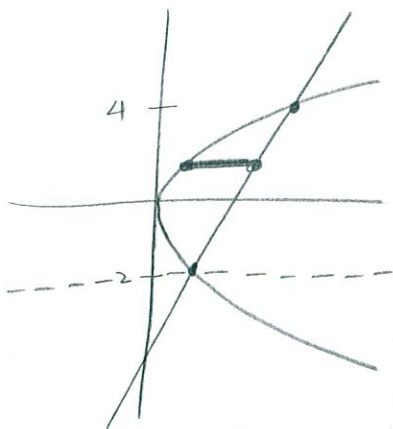
① POINT EACH

$$\begin{aligned} & \int_{-2}^1 (4x^2 - (x+3)^2) dx \\ & + \int_{-1}^2 ((x+3)^2 - 4x^2) dx \\ & = \left| \int_{-2}^1 (3x^2 - 6x - 9) dx \right| + \left| \int_{-1}^2 (-3x^2 + 6x + 9) dx \right| \\ & = \left| (x^3 - 3x^2 - 9x) \right|_{-2}^1 + \left| (-x^3 + 3x^2 + 9x) \right|_{-1}^2 \\ & = ((-1 - 3 + 9) - (-8 - 12 + 18)) \\ & + ((-8 + 12 + 18) - (1 + 3 - 9)) = 5 + 2 + 22 + 5 \\ & = \underline{34} \end{aligned}$$

The region bounded by $y^2 = 4x$ and $y = 2x - 4$ is revolved around the line $y = -2$.

SCORE: ____ / 12 PTS

Find the volume of the resulting solid. (NOTE: Your final answer must be a number, not an integral (or sum of integrals).)



$$x = \frac{1}{4}y^2 \quad x = \frac{1}{2}y + 2$$

$$\frac{1}{4}y^2 = \frac{1}{2}y + 2$$

$$y^2 = 2y + 8$$

$$y^2 - 2y - 8 = 0$$

$$(y - 4)(y + 2) = 0 \rightarrow y = 4, -2$$

$$\begin{aligned} & \textcircled{2} \int_{-2}^4 2\pi \left(\frac{1}{2} \right) (y - 2) \left(\frac{1}{2}y + 2 - \frac{1}{4}y^2 \right) dy \\ &= 2\pi \int_{-2}^4 \underbrace{(2+y)}_{\textcircled{1/2}} \underbrace{\left(2 + \frac{1}{2}y - \frac{1}{4}y^2 \right)}_{\textcircled{2}} dy \\ &= 2\pi \int_{-2}^4 (4 + 3y - \frac{1}{4}y^3) dy \\ &= 2\pi \left(4y + \frac{3}{2}y^2 - \frac{1}{16}y^4 \right) \Big|_{-2}^4 \\ &= 2\pi (16 + 24 - 16 - (-8 + 6 - 1)) \\ &= 2\pi (24 - 3) \\ &= 54\pi \end{aligned}$$

TALK TO ME
IF YOU USED
WASHER METHOD
(dx)